Double Pendulum Power

Method for Extracting AC Power from a Mechanical Oscillator

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1 Introduction

The idea is that of a double pendulum where an oscillating torque is created in the fixture point (black in figure below) by utilizing the centrifugal force of a rotating outer pendulum. We will show that this force transfers kinetic energy to the inner pendulum mass and that this energy can be useful. After the energy needed to set the outer pendulum in motion is supplied, only energy to overcome friction is needed to keep it in motion. The centrifugal force of the outer pendulum will then make the inner pendulum oscillate at a frequency and amplitude defined by the Euler Lagrange equations that we will examine numerically using the Runge Kutta method.

This is possible because we are dealing with an open system. The double pendulum needs to be properly fixed otherwise the vibrations will just continue to the surroundings or being damped and turned into heat and possibly wear out the fixture materials. So in that sense a tiny movement of that heavy object is the other side of the energy equilibrium we are dealing with.

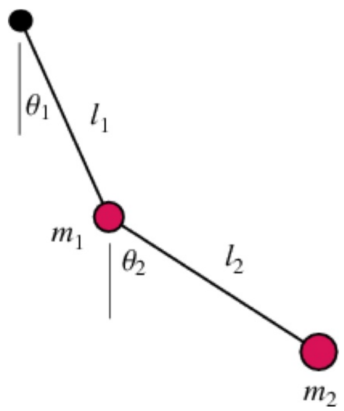

Figure 1: double pendulum

2 The Physics

First consider the well known Euler Lagrange equations for the double pendulum.

$$(m_1+m_2)l_1\ddot{\theta}_1+m_2l_2\ddot{\theta}_2cos(\theta_1-\theta_2)+m_2l_2\dot{\theta}_2^2sin(\theta_1-\theta_2)+g(m_1+m_2)sin\theta_1 = 0 \quad (1)$$

$$m_2l_2\ddot{\theta}_2 + m_2l_1\ddot{\theta}_1cos(\theta_1-\theta_2) - m_2l_1\dot{\theta}_1^2sin(\theta_1-\theta_2) + m_2gsin\theta_2 = 0 \quad (2)$$

To be able to solve these equations using the Runge Kutta method we elaborate the equations for the angular accelerations respectively; $\ddot{\theta}_1$ and $\ddot{\theta}_2$.

$$\begin{aligned}\ddot{\theta}_1 =& \frac{-g(2m_1+m_2)sin\theta_1 - m_2gsin(\theta_1-2\theta_2)}{l_1(2m_1+m_2-m_2cos(2\theta_1-2\theta_2))} \\ &- \frac{2sin(\theta_1-\theta_2)m_2(\dot{\theta}_2^2l_2+\dot{\theta}_1^2l_1cos(\theta_1-\theta_2))}{l_1(2m_1+m_2-m_2cos(2\theta_1-2\theta_2))}\end{aligned} \quad (3)$$

$$\ddot{\theta}_2 = \frac{2sin(\theta_1-\theta_2)(\dot{\theta}_1^2l_1(m_1+m_2)+g(m_1+m_2)cos\theta_1+\dot{\theta}_2^2l_2m_2cos(\theta_1-\theta_2))}{l_2(2m_1+m_2-m_2cos(2\theta_1-2\theta_2))} \quad (4)$$

Finally we set up the equation for the power needed to either increase (accelerate) or decrease (decelerate) the speed (kinetic energy) of the inner pendulum mass (m_1) with the pendulum arm (l_1). This is the power that we propose could be partially used for generating useful energy.

$$P_{m_1} = m_1l_1^2\ddot{\theta}_1\dot{\theta}_1 \quad (5)$$

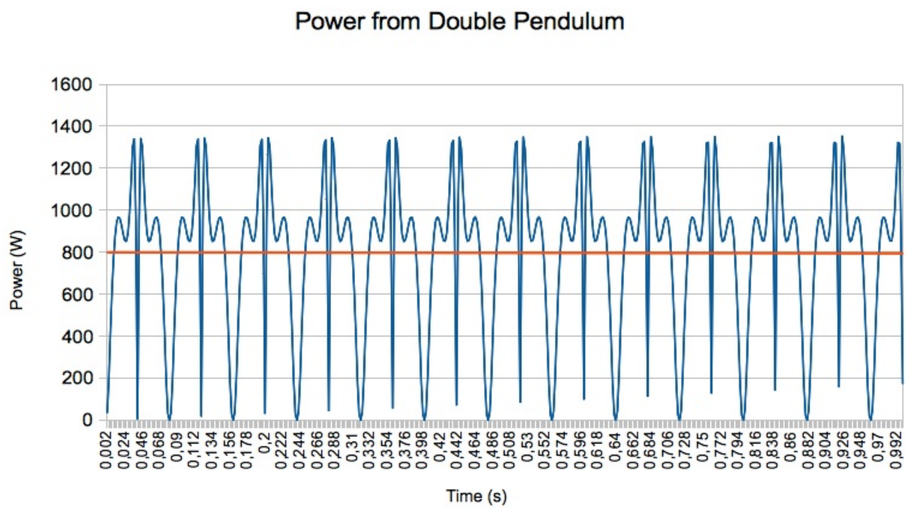

Figure 2: Power from Double Pendulum as a function of time

The Runge Kutta method is used to solving the equation numerically. We use the following input data: $m_1 = 1$kg, $m_2 = 1$kg, $l_1 = 0.1$m, $l_2 = 0.1$m, $\dot{\theta}_2 = 100$ rad/s (i.e. the initial speed of rotation of the outer pendulum is ≈ 16 Hz which equals an initial kinetic energy of $E_k = 50$J). We also assume rotation is in the horizontal level so that $g = 0$ and that there is no friction. As we can se in the graph the outer pendulum is constantly transferring energy to the inner pendulum mass by use of centrifugal force and vice versa. The result is that the the both the pendulum masses are constantly accelerating/decelerating (oscillating), without any more input of energy. And as Newton showed, accelerating mass is a manifestation of energy: $E_{energy} = m_{mass} a_{acceleration} s_{distance}$.

We thereby conclude that by setting the outer pendulum in motion with only $E_k = 50$J we continuously either accelerate or decelerate the inner pendulum mass with a power averaging $\bar{P}_{m_1} \approx 800$W.

What we want to emphasize here, is that the fact that speed and acceleration is vector based but energy and power is scalar, makes it possible to make this power useful. The schematic below outlines the principle of this line of thought.

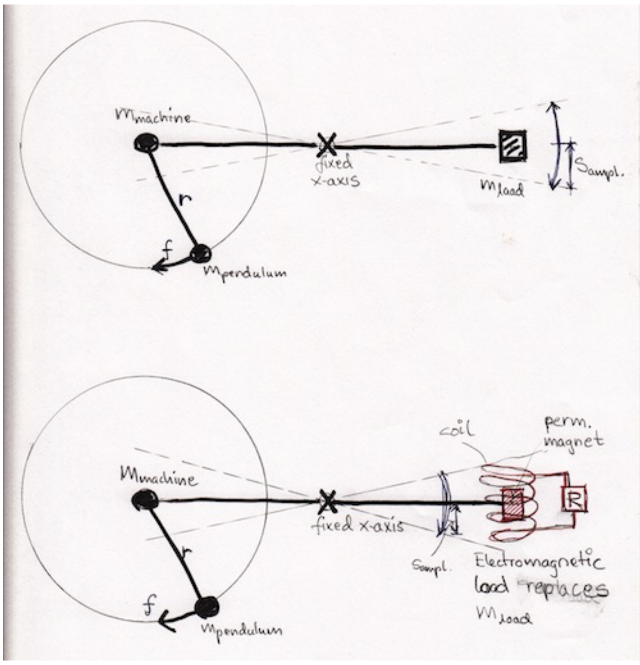

Figure 3: Double Pendulum with extra load (mass or electromagnetic)

What we do is to simply assume that the power used to accelerate or decelerate the inner pendulum mass can be useful if we replace the mass with a generator. In theory all of the replaced inner pendulum mass could be useful, but in reality it is probably not the case because of design and construction issues. Thus the distinction between the useful m_{load} and $m_{machine}$ in the schematic above.

The efficiency of the generator concept could be described by the following equation. As you can see a lightweight machinery of oscillating parts (mass) is always a good idea.

$$\eta_{efficiency} = \frac{m_{load}}{m_{load} + m_{machine}} \tag{6}$$

3 The Case of the Milkovic Pendulum

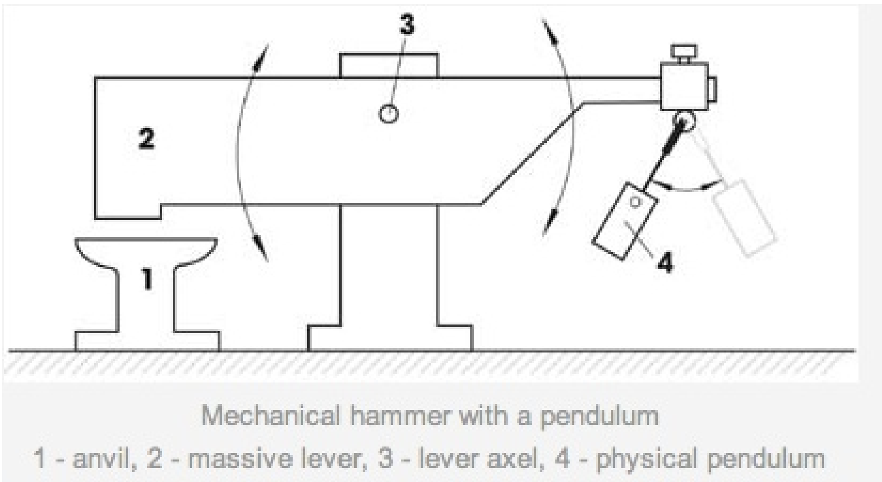

Figure 4: Milkovic Pendulum (from http://www.pendulum-lever.com)

The Milkovic pendulum is a handdriven pendulum that could be used to pump water. We've been examining a pendulum with our numerical model with the following input parameters: (2)$m_1 = 10$ kg, (4)$m_2 = 10$ kg, $l_1 = 0.5$ m, $l_2 = 0.3$ m, $\theta_2 = 2$ rad (i.e. we lift the outer pendulum before we let it go, thereafter only affected by gravity). Lets have a look at the characteristics of the movements of the pendulums in a fricionless environment.

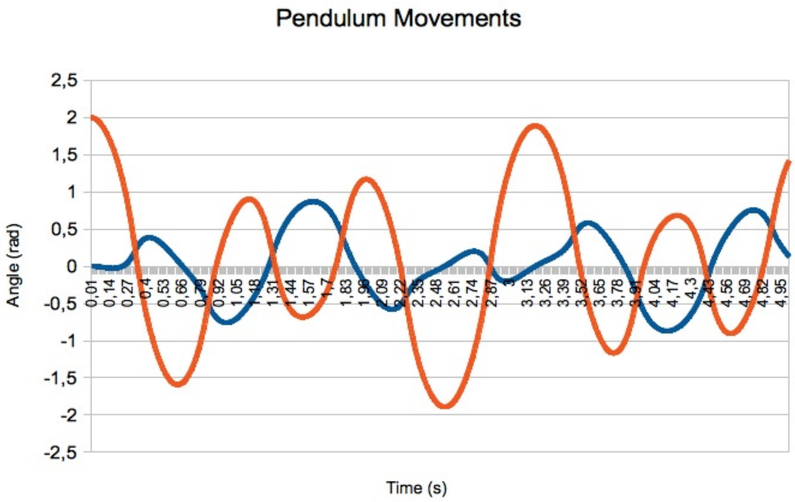

Figure 5: Pendulum movements, m_1/massive lever (blue), m_2/pendulum (red)

Lets assume that half of the power produced by the pendulum is used to move the inner pendulum arm (i.e. "massive lever" machinery) and the other half is pumping water. Using Runge Kutte we calculate the prower produced as a function of time.

$$P_{m_1} = \eta_{m_1} m_1 l_1^2 \ddot{\theta}_1 \dot{\theta}_1 \tag{7}$$

Where η_{m_1} is the fraction of the mass that accounts for the usable power. In this case $\eta_{m_1} = 0.5$.

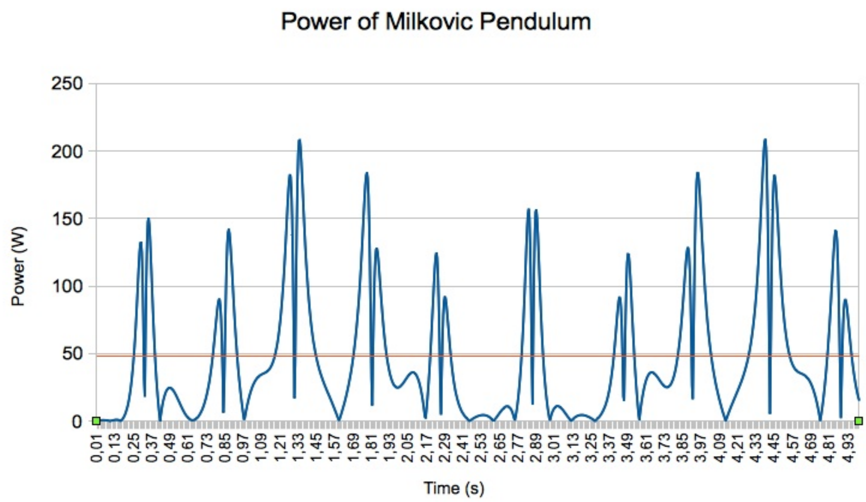

Figure 6: Pendulum power at 50% efficiency averaging about 50W

As we can se the pendulum is producing an average of about 50W with no input except for the $E_{in} = 42$ J used to lift the pendulum initially (2 rad). It will keep swinging in a frictionless environment. It will however swing in a more or less unpredictable manner depending on the initial conditions. As we can see a considerable amount of the power will be delivered in the form of spikes.

Of course there are other issues about the construction. As an example Milkovic uses a much heavier machinery which will decrease efficiency since so much power is used to move/oscillate the actual inner pendulum arm (massive lever). We suspect Milkovic is doing this to control the output/input movements and amplitude. As an example; if the inner massive lever is increased to 100 kg (i.e. a tenfold increase) the output power will average about 27W at 50% efficiency. A reduction of 46%, but with a more controllable output amplitude.

The conclusion is that by pushing the pendulum to an equal angle at each oscillation in order to overcome friction, it is possible to continuously produce output power much greater than the power needed to push the pendulum at low enough friction. The power will however be delivered in a somewhat unpredictable manner.

4 Generating Power

This report is however not so interested in handdriven pendulums as in the possibility to build a generator that utilize the power in a automized manner and generating electrical power at higher frequencies. We've already looked at an example which rotates horizontally at 16 Hz producing an average of 800W.

The interdependencies between the two pendulums continuously exchanging kinetic energy as they oscillate is complex. The two pendulums transfer momentum between themselves in both directions as described by the Euler Lagrange equations earlier. If we try to force the outer pendulum into a certain speed or movement, we will no doubt disturb the inner pendulum as it transfer momentum back. Constant rotation of the outer pendulum will simply not do.

At this point we continue by examine the motions and power resulting from certain initial values and leave the control problem for later.

Below we will have a look at a pendulum with the following input parameters: $m_1 = 1$kg, $m_2 = 0.1$kg, $l_1 = 0.2$m, $l_2 = 0.1$m, $\dot{\theta}_2 = 314$ rad/s (initial speed of rotation is 50 Hz).

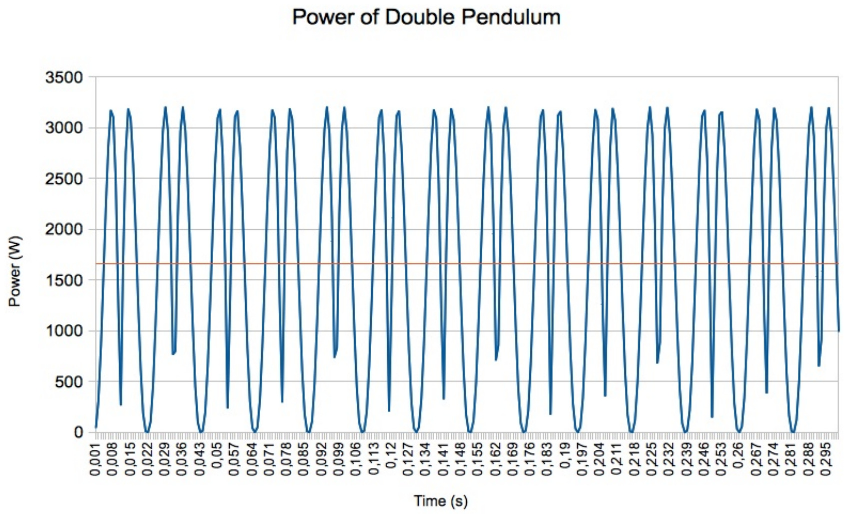

Figure 7: Pendulum power from 50 Hz input, averaging about 1600 W

The most important fact about this pendulum is the considerable power generated from such a small device. A 1kg load on a 0.1kg pendulum with levers of 0.2m and 0.1m respectively continuously generates 1600 W from only 49 J input. By now you probably realize why this is important. It could be possible to build really compact generators using this method if we overcome the engineering problems. For construction and material strength purposes we also note that the rotating pendulum applies an equally large static centrifugal force on the fixture in the direction of the inner pendulum arm. This engineering problem can be solved by using two synchronized outer pendulums as described in the next section.

5 Construction Schematic

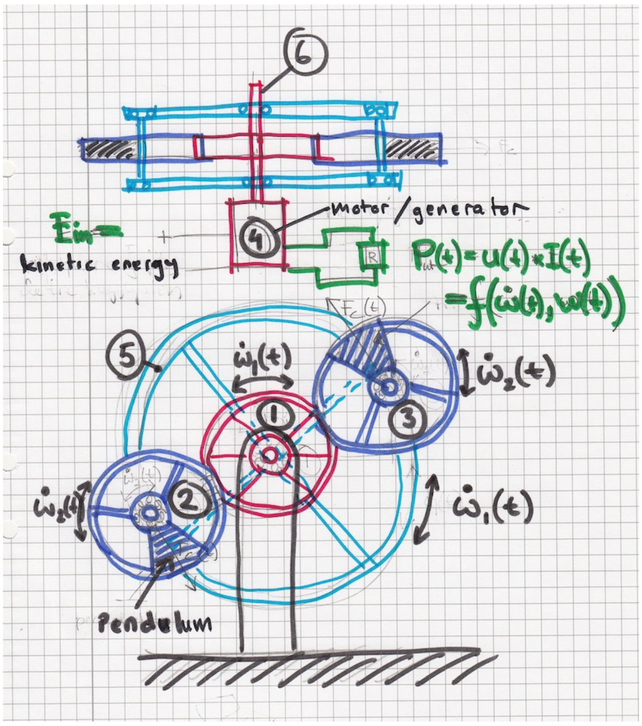

Figure 8: Construction schematic

This is a simple schematic of a generator using what we've described. On the input side we have a motor attached to a driving cogwheel, which rotates the two unbalanced cogwheels (i.e. pendulums). These cogwheels are synchronized so that all forces in the direction to and from the fixed axis is cancelled out at all times, which will reduce the stress on the fixture. The unbalanced cogwheels are mounted on a frame that transfers momentum to the generator.

- (1), (2) and (3) are cogwheels, where (2) and (3) are unbalanced. They can be considered pendulums.
- (4) is a motor/generator
- (2) and (3) are mounted on a "frame" (5) which rotates frictionless on the axis (6).
- The motor/generator is fixed on cogwheel (1) so that it can either drive or receive energy/momentum to/from the system.
- The motor/generator is securely mounted on the floor/surroundings.

We then suggest the following line of thought.

1. The frame (5) with the cogwheels (2) and (3) is fixed so that it can not rotate around axis (6).

2. The cogwheels are set in motion at the desired rotation speed with the motor. The frame is still fixed.

3. The motor drive is disconnected, now the motor acts as a generator. Still there is no load so the generator rotates frictionless.

4. The frame (5) fixation is disconnected sot that it now rotates/oscillates freely around axis (6) and transfers momentum to the generator (4).

5. Load is added on the generator.

If the machine was frictionless we would be able to extract power continuously. However this is of course not the case and a system for adding power to overcome friction is needed. This is the tricky part because there is a constant feedback of momentum coming from the pendulums (2) and (3) to the driving cogwheel (1). This means that it will not be possible to add a constant driving momentum on cogwheel (1) since this momentum simply will work the machine in the wrong direction half of the time.

Therefore, we need to design a system that pulses the correct amount of energy/momentum to the driving cogwheel (1) with the right timing. Obviously this would be in the opposite direction of friction, i.e. always in the direction of the current acceleration. Here is a graph showing the accelerations from the pendulums as a function of time.

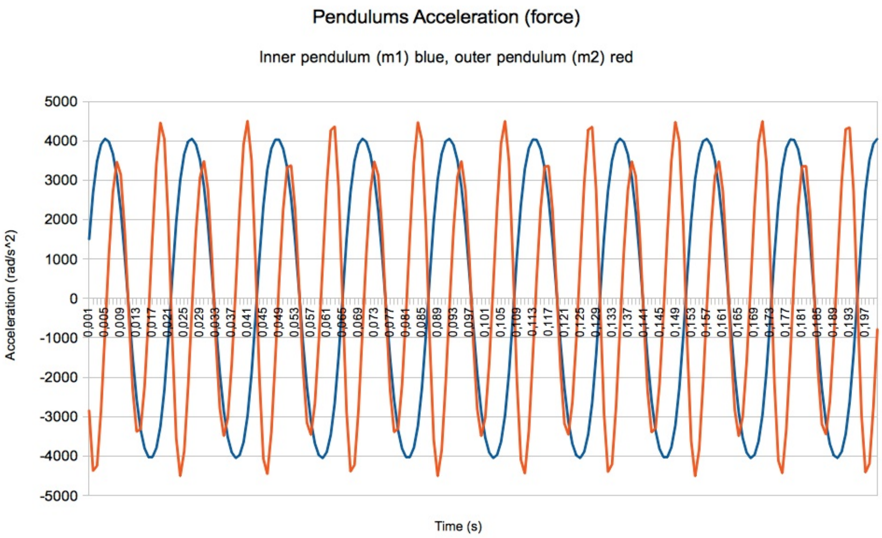

Figure 9: Pendulum accelerations. M1 is blue and M2 is red

From this graph we can conclude that it is possible to use the output from the inner pendulum, i.e. the generator output, as an input parameter for the pulsed input power on the driving cogwheel. Our suggestion is to add a pulsed input power for maybe 1/10 of the time of the initial frequency of rotation (i.e. 1/500 s), in the opposite direction of the initial rotation each time the output power is zero (0), which is twice per oscillation. It's actually a simple case of resonance. The analogy for this is obvious; like pushing a playground swing or for that matter, the Milkovic pendulum.

6 Building a Household Generator

If we, as an example, want to build a household generator we could use two (or as many as we need) 0.1 kg pendulums with a radius of 0.1 m, mounted on a frame with the radius 0.2 m. The pendulums are set in initial rotation at 50Hz. If the weight of the cogwheels and levers are assumed to be positioned at the cogwheel/pendulum center, and if this "machine weight" is 1 kg per pendulum at a efficiency of 50%, the output will be approximately 3.2 kW of AC power. The output power distribution will look like in figure 7 (but twice the amplitude) and the electrical output something like in the figure 10 below.

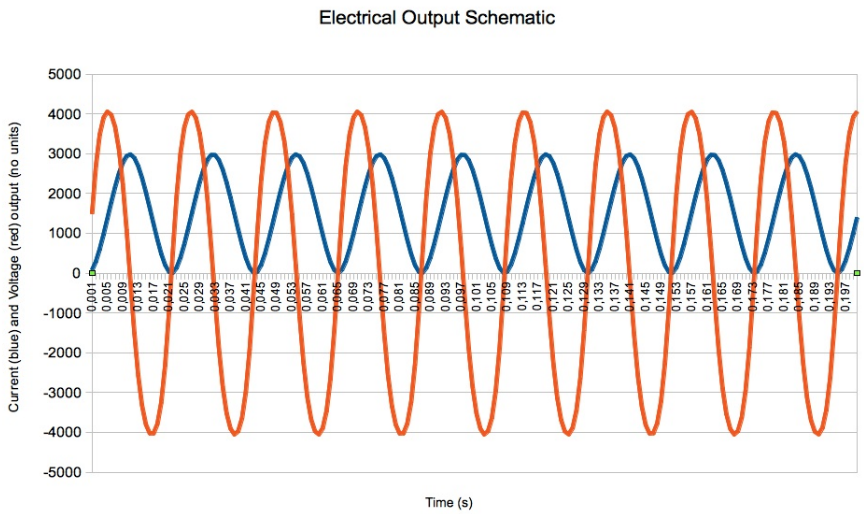

Figure 10: Electical output schematic. Voltage (blue), current (red)

The equations for the output power is as follows. $m_{MechLoad}$ describes the productive load/momentum, which is not the same as m_1 that is the combined mass of the load and the unproductive weight of the cogwheels and frame (i.e. machine).

$$P(t)_{output} = m_{MechLoad} r^2_{frame} \omega'_{AccOfFrame} \omega_{SpeedOfFrame} \tag{8}$$

From Kirchoff we know that the voltage over a coil and a resistive load is described as follows.

$$Li' + iR_{load} = 0 \tag{9}$$

Consequently we get for the complete generator with electrical output.

$$P(t)_{output} = Lii' = i^2 R_{load} = m_{MechLoad} r_{frame}^2 \omega'_{AccOfFrame} \omega_{SpeedOfFrame} \tag{10}$$

As we can see, the current i_{output} correlates directly with the speed of rotation ω and the voltage $u_{output} = Li'$ correlates directly with the acceleration ω'.

7 Conclusions

This document shows that it is possible to utilize the constant force acting through the arm of a pendulum in motion. The force is a function of speed of rotation (ω) but results in an acceleration of mass, i.e energy. The energy will manifest itself in the form of vibrations. We then make the connection between these vibrations and the characteristics of AC current and realize that it is exactly what we are looking for.
The power extracted is a function of the frequency of rotation by the power of three ($mr^2\omega'\omega$ or Lii'). This also explains the extreme power of vibrations, for example in buildings, bridges and other constructions. Even minute imbalances in an engine gets the whole car to vibrate. And so on, and so on.

We also show the theory behind the dynamics and point out the fact that it is an open system that does not violate any classic Newtonian laws but can be described using a numerical solution (Runge Kutta) of the Euler Lagrange equations of the double pendulum.

With this foundation of controlled oscillations directly converted into AC power, simple motors can be built in any small village workshop everywhere around the globe. Help can be supplied with construction and motor design if needed. We emphasize the importance of design, and that the construction for transfer of power need to be lightweight i relation to the weight of the pendulum.

As our hero Nikola Tesla famously said: *"If you want to find the secrets of the universe, think in terms of energy, frequency and vibration."*